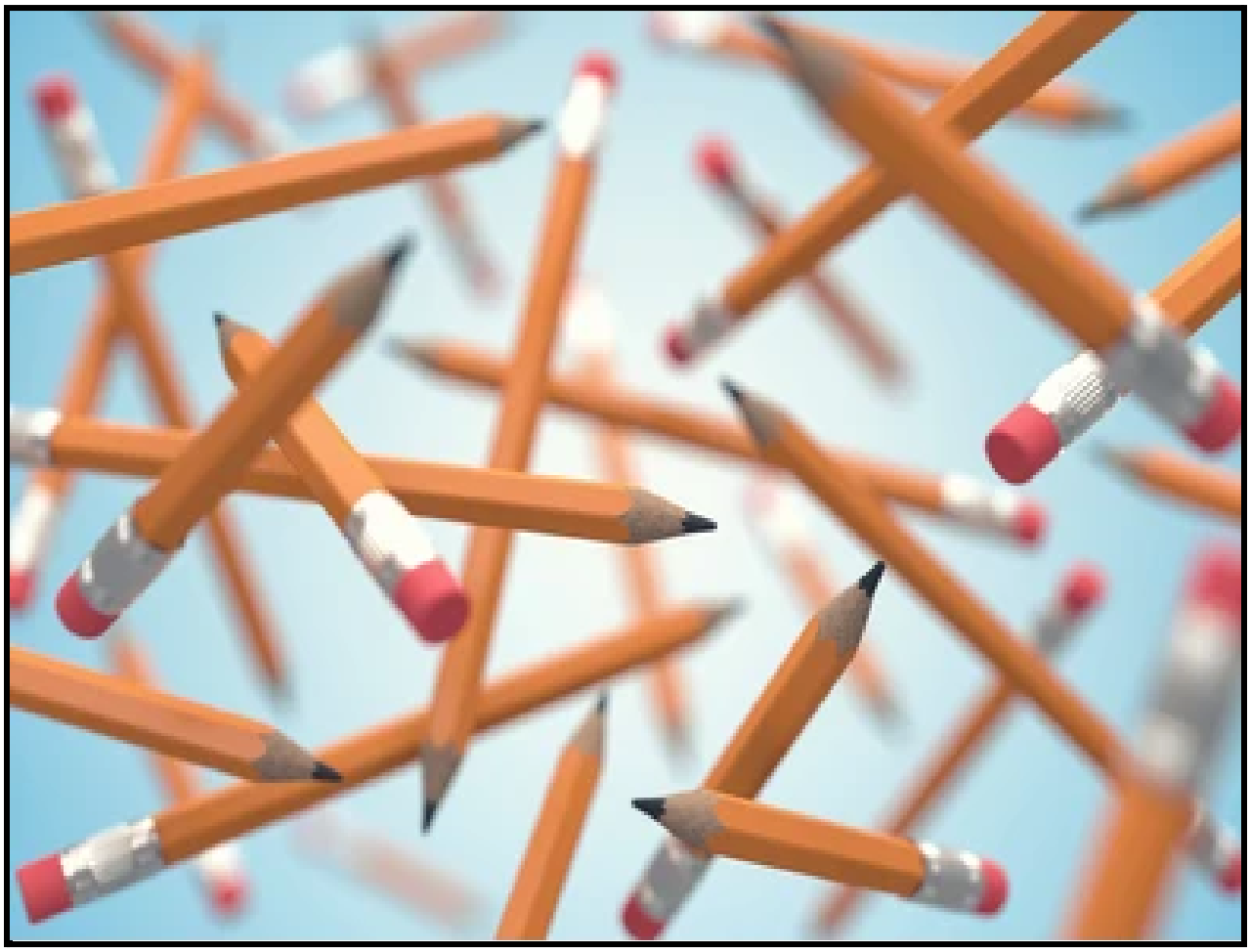
Effect of Pencil Type on Distance Flown



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Introduction:

Pencils are often used for writing, erasing, and creating monochromatic art easily. However, they also are cheap, light, and aerodynamic projectiles. It is not uncommon for people, especially children, to launch one of these projectiles off of a table by utilizing a small amount of force. For those unfamiliar with this activity, a diagram of the pencil-launching maneuver is on page 4.

As a pencil-launching enthusiast myself, I decided to conduct an experiment to determine whether mechanical pencils or graphite pencils flew farther. The results of this experiment are based on principles such as Newton's third law and air resistance, so there is an academic application as well. In this experiment, I hope to find the better pencil type for launching and thus increase my enjoyment. Future experiments may investigate the optimal pencil length or shape too.

As for my hypothesis, I believe that if graphite pencils are launched, they will fly farther than mechanical pencils. I arrived at my hypothesis as more weight in a more uniform distribution results in overall decreased air resistance, and graphite pencils fit that description.

Experiment Design:

The experimental units are the 60 pencils, 30 of which are graphite pencils and the other 30 are mechanical pencils.

The order in which the pencils are tested is randomized to minimize confounding variables in the following way:

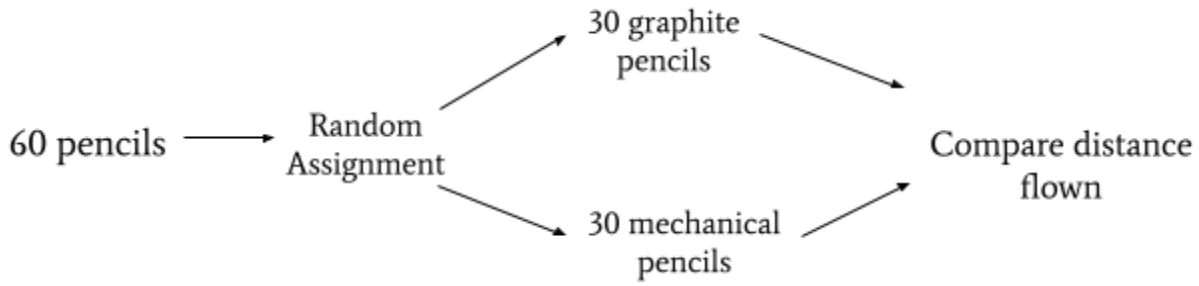
1. Randomly select 30 graphite pencils and 30 mechanical pencils
2. Number the graphite pencils from 1 to 30 and number the mechanical pencils from 31 to 60.
3. Run *"sample(1:60, 60, replace = FALSE)"* in R.
4. Going through the resulting numbers in order, conduct a trial on the corresponding pencil.

Since this experiment is not on people, blindness and double blindness are not applicable. Blocking is also not applicable in this experiment.

Experimental Design Table:

Experimental Design Table		
Title: Effect of Pencil Type on Distance Flown		
Hypothesis: If graphite pencils are launched instead of mechanical pencils, then they will fly farther because mechanical pencils are heavier and their weight is distributed unevenly.		
Explanatory Variable: The type of pencil used (graphite or mechanical)		
Treatments:	Graphite	Mechanical
Repeated trials:	30 repeated trials	30 repeated trials
Response Variable: Distance flown in inches from the launcher to the tip of the pencil		
Constants: ● Pencil length suspended off of the edge (2.5 in.) ● The Pencil-Launching Apparatus (described on the next page) ● The Tape Measure ● The orientation of the pencil when launched		

Experimental Design Diagram:



Materials:

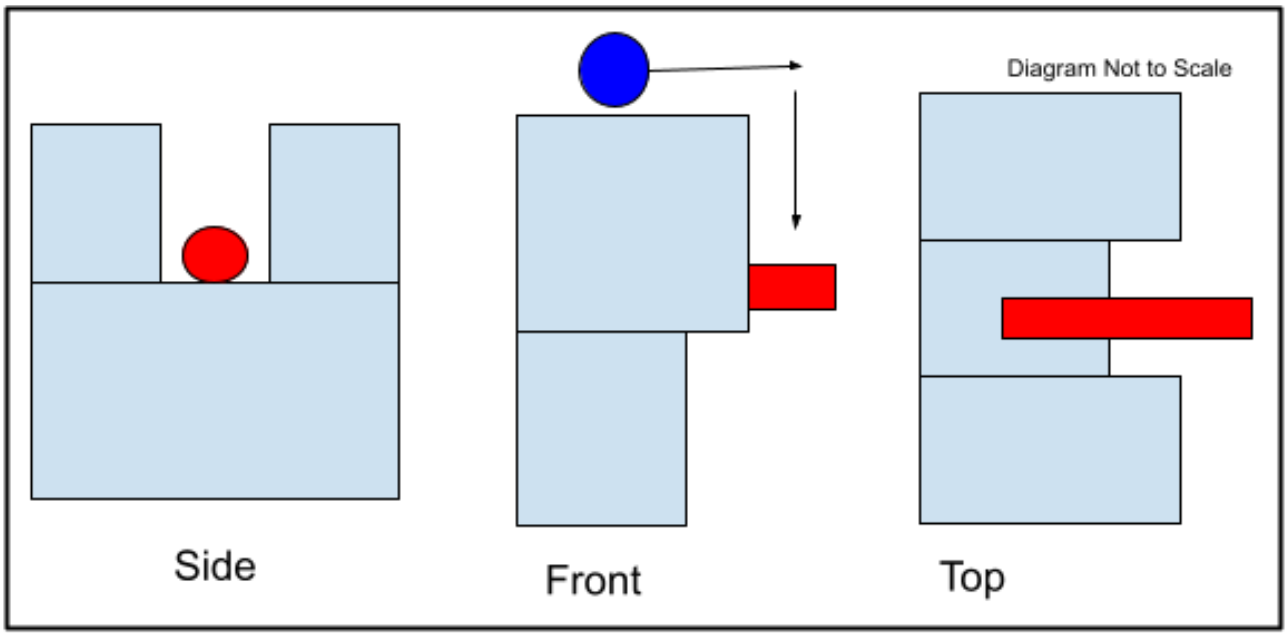
- 30 mechanical pencils
- 30 graphite pencils
- 1 Pencil-Launching Apparatus
    - 1 table or a flat raised surface
    - 2 solid blocks 7.5 inches tall (such as a stack of books)
    - 1 Object
        - Roller (picture below)
    - One piece of paper (to use as reference when launching)
- 1 tape measure (for measuring distance)

The Object:

10.5 in. long from the ends of the middle cylinder

The Pencil-Launching Apparatus:

A diagram of the apparatus and the basic launching maneuver. is below. The light blue blocks represent the solid objects; the table and the books. The red block shows how the pencil is placed. Finally, the dark blue block is the object rolled off of the light blue blocks and launches the pencil as it falls off of the edge. In practice, a swift hand or arm fulfills the function of the dark blue block, but that would be a confounding variable in an experiment.



Procedure:

1. Construct the Pencil-Launching Apparatus as shown above. Acquire 30 new graphite pencils and 30 new mechanical pencils.
2. Generate a Random Order of testing each type of pencil for 60 trials.
3. For each trial:
    a. Arrange the pencil in the correct position with 3 inches suspended off of the edge.
    b. Carefully roll the object off of the blocks so that it has minimal forward velocity.
    c. Measure the distance from the launch point to the farthest point of the pencil.
    d. Record the data in a table with two columns and 30 rows; one row for each trial and one column for each treatment (graphite or mechanical pencil).
5. Once all 60 trials have been completed and the table is filled, calculate the mean of each column of the table and reach a conclusion as to which type of pencil flies farther.

The Experiment:

Pencil Set:

Below are pictures of the pencils I used. Since there were more than 30 for each of them, I used R to randomly select some to remove so I could have 30 pencils of each type.

Graphite (x2):

Mechanical:

Final Set of Pencils (numbered horizontally):

Pencil-Launching Apparatus:

This is my pencil-launching apparatus. I used books for the blocks and made them have equal height. The paper at the bottom is used to position the pencil with the correct length suspended off of the edge. I also spaced out the books so that the roller would be easier to align. Underneath the table, I secured a tape measure to measure the distance easily.

Data:

Graphite:

Pencil #	Distance (in.)	Pencil #	Distance (in.)
1	32.5	16	26.3
2	30.8	17	21.2
3	36.7	18	32.3
4	25.6	19	31.3
5	37.0	20	45.3
6	34.2	21	44.1
7	53.1	22	33.5
8	21.7	23	21.3
9	18.1	24	26.3
10	51.4	25	24.4
11	25.0	26	23.1
12	30.1	27	49.7
13	56.4	28	34.2
14	30.0	29	26.9
15	44.3	30	25.0

Mechanical:

Pencil #	Distance (in.)	Pencil#	Distance (in.)
31	15.9	46	28.7
32	19.5	47	27.7
33	25.7	48	24.3
34	22.3	49	23.4
35	16.9	50	20.4
36	35.1	51	23.5
37	19.2	52	21.9
38	14.5	53	23.8
39	24.3	54	38.0
40	31.3	55	28.1
41	25.3	56	29.1
42	45.5	57	35.7
43	40.6	58	18.6
44	21.6	59	12.3
45	34.3	60	40.6

Order of Testing:

10 29 21 55 54 38 46 1 11 27 52 56 8 41 59 51 49 39 58 40 14 15 26 48 4 37 32 17 9 22 16 35 53 50 24 57 12 19 3 25 36 47 2 60 28 13 30 20 6 33 5 18 43 31 45 34 42 44 7 23

There are no derived quantities.

Data Analysis:

All numbers have inches as units except sample size, which is in pencils.

	Graphite Pencils	Mechanical Pencils
Sample Mean	33.06	26.27
Sample Standard Deviation	10.40	8.31
Minimum	18.10	12.30
Lower Quartile	21.15	20.70
Median	31.05	24.30
Upper Quartile	36.92	30.75
Maximum	56.40	45.5
Outliers	56.40	N/A
Sample Size	30	30

Empirical Rule Estimates:

One Standard Deviation (68%):

Graphite: 63.33% (~5% away from Empirical Rule estimate)

Mechanical: 66.67% (Close to Empirical Rule estimate)

Two Standard Deviations (95%):

Graphite: 96.67% (Close to Empirical Rule estimate)

Mechanical: 96.67% (Close to Empirical Rule estimate)

Three Standard Deviations (99.7%):

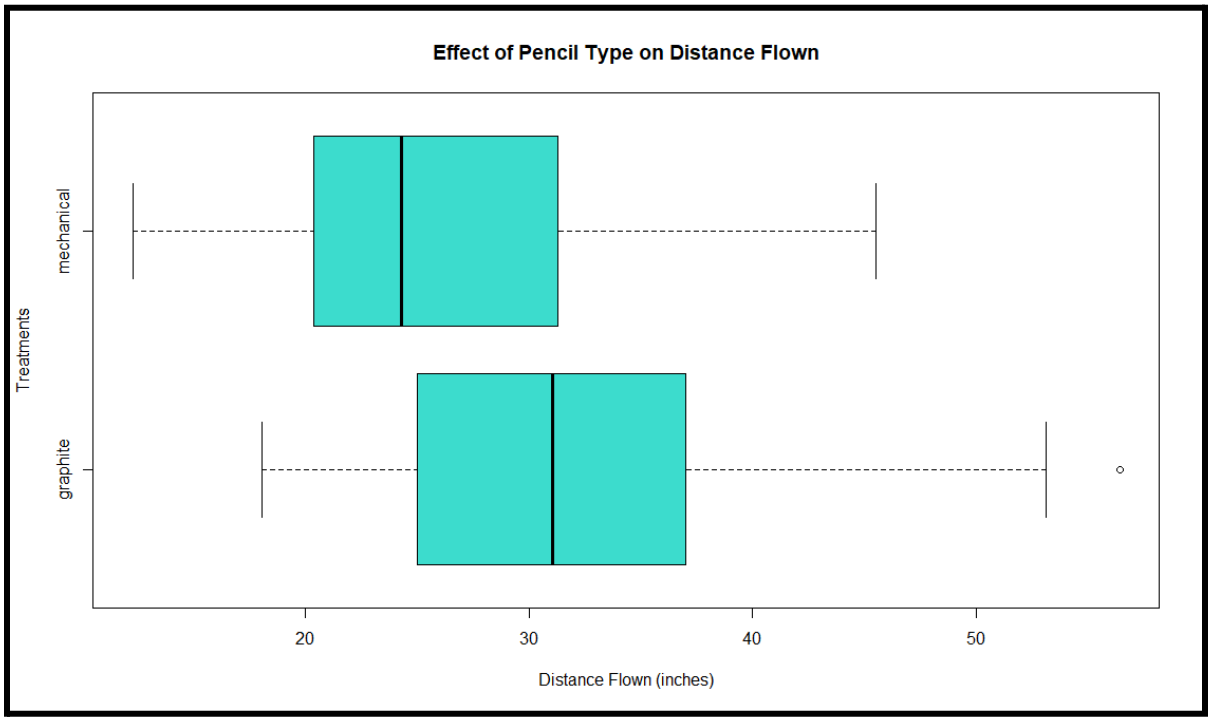
Graphite: 100% (Close to Empirical Rule estimate)

Mechanical: 100% (Close to Empirical Rule estimate)

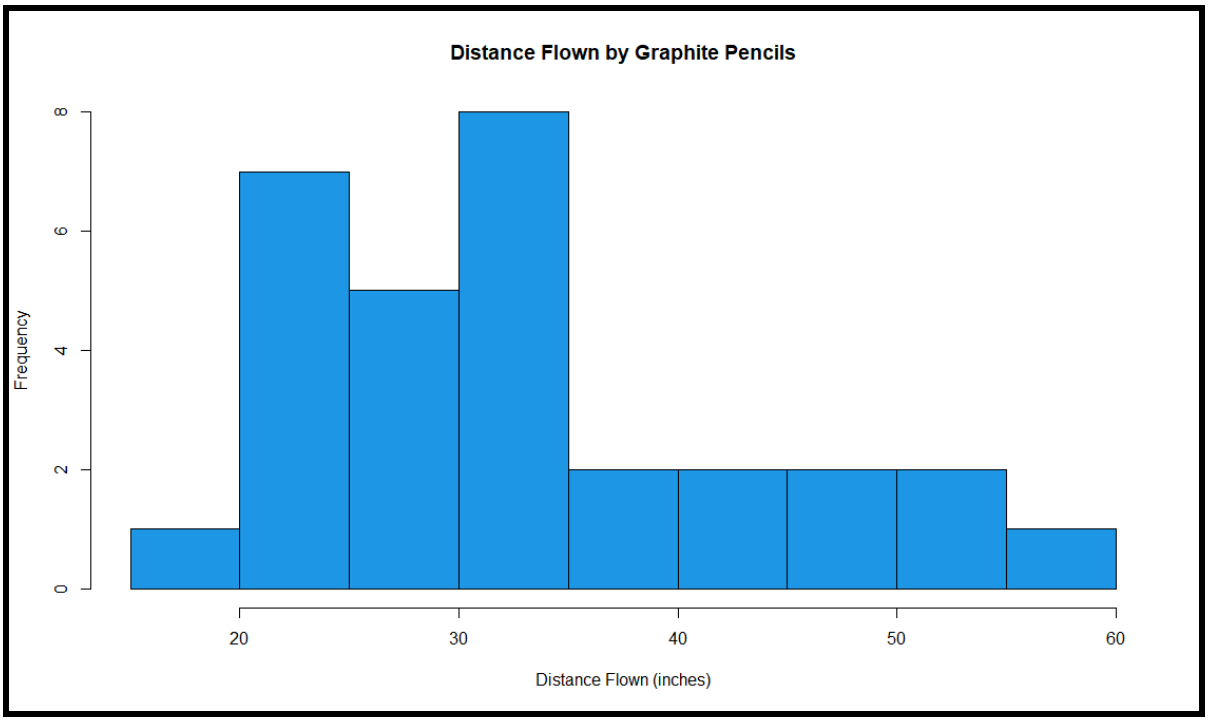
This indicates that while both pencils are mostly normally distributed, mechanical pencils are much more normal than graphite pencils.

Graphical Displays:

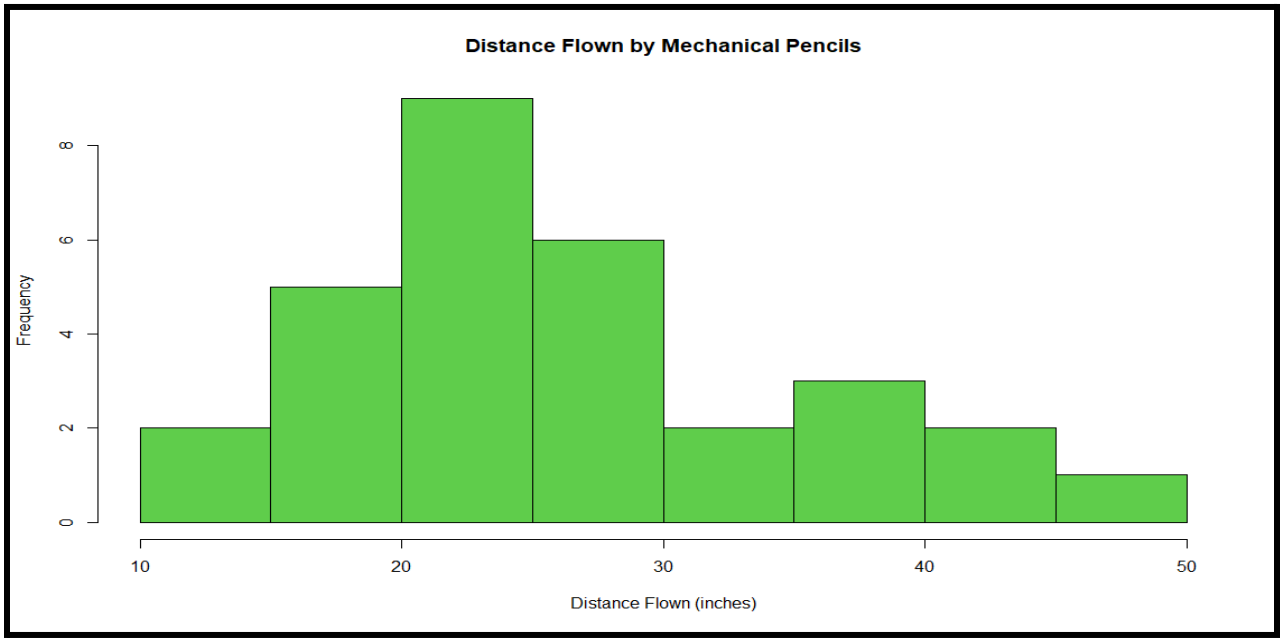
Comparison between Graphite and Mechanical pencils:



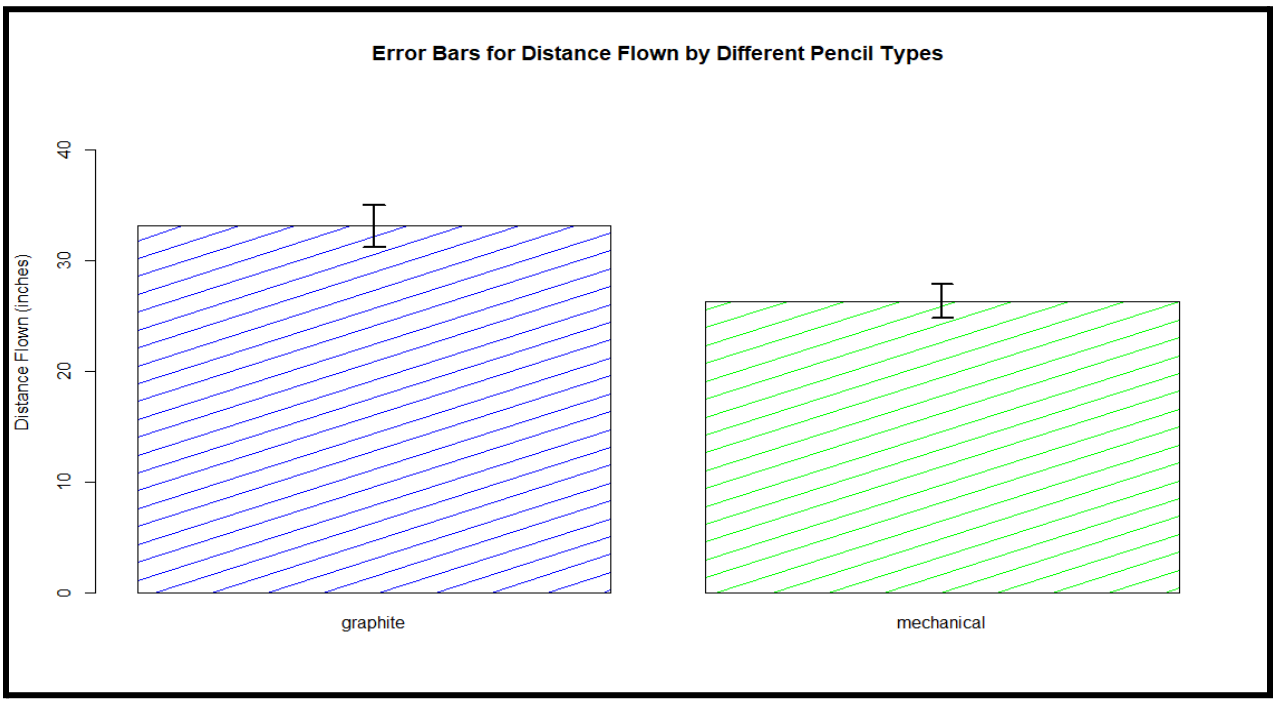
Histogram of Graphite Pencils:



Histogram of Mechanical Pencils:



Error Bars:



Standard Error of Mean:

Graphite: 1.899 inches - 31.16 to 34.96 inches

Mechanical: 1.517 inches - 24.76 to 27.79 inches

Description:

The data above compares the distance flown between graphite and mechanical pencils. For graphite pencils, the data seems to be right skewed and bimodal with peaks at 22 and 32 inches. For mechanical pencils, the data is slightly less right skewed and unimodal, with a peak around 25 inches. In graphite pencils, there is a gap between distances between 38 and 44, but the data is spread out evenly otherwise. In mechanical pencils, there are a few small gaps to the right of the median. There is one outlier for graphite pencils on the right; 56.4 inches. There are no other outliers on both graphs.

The centers of both distributions are calculated through the mean and the median. The mean distance flown for graphite pencils is 33.06 inches while the median is 31.05. The mean distance for mechanical pencils is 26.27 and the median is 24.30. Since the median is less than the mean in both cases, we can infer that both datasets are right skewed. However, the mean is only about 2 inches above the median in either, so it is only a slight right skew. Calculating the standard deviation as well as the range will give an idea of the spread of the distributions. The standard deviation for graphite pencils is 10.4 inches and the standard deviation for mechanical pencils is 8.31 inches. The ranges are 38.3 and 33.2 inches for graphite and mechanical pencils, respectively. These values indicate that graphite pencils are slightly more spread out than mechanical pencils.

In conclusion, both graphite and mechanical pencils are similar in shape and spread, but graphite pencils are slightly more bimodal, right skewed, and spread out. Also, graphite pencils have one outlier on the right while mechanical pencils have none. The largest difference between the two distributions is their centers. The mean and the median of graphite pencils are about 7 inches higher than the respective mean and median of mechanical pencils. This indicates that the difference between the two pencil types might be statistically significant. In addition, the error bars on the mean do not overlap, meaning that we cannot be sure that the difference is not statistically significant. Therefore, our hypothesis is likely correct.

To answer the question regarding normality, we know that our sample sizes are sufficiently large, as they are both 30. Independence is also satisfied as 30 is definitely less than 10% of the total number of pencils. Our empirical rule estimates show that the distribution of mechanical pencils is mostly normally distributed while the distribution of graphite pencils is less normal. This is likely due to the fact that the graph of graphite pencils is bimodal. However, the population is still mostly normal, so the assumption of normality is definitely plausible.

Test of Significance:

State:

- μ_G = mean distance flown by graphite pencils
- μ_M = mean distance flown by mechanical pencils
- n_G = sample size for graphite pencils
- n_M = sample size for mechanical pencils
- $H_0 : \mu_G - \mu_M = 0$
- $H_A : \mu_G - \mu_M > 0$
- $\alpha = 0.05$

Plan:

- Random: The pencils and the order in which they were selected were random.
- Normal: $n_G \geq 30$ and $n_M \geq 30$
- Independence: Safe to assume that n_G and n_M are less than 10% of the total population of graphite and mechanical pencils.

Do:

In R run "*tsum.test(mean.x = 33.06, s.x = 10.4, n.x = 30, mean.y = 26.27, s.y = 8.31, n.y = 30)*".

- Test statistic: 2.794
- Degrees of freedom: 55.307
- p-value = 0.0036

Conclude:

Since the p-value is less than α, we reject the null hypothesis. There is sufficient evidence to say that graphite pencils fly farther than mechanical pencils on average.

Conclusion:

The purpose of this experiment was to determine the optimal pencil type for launching. Graphite and mechanical pencils have slightly different aerodynamics, but I found that it corresponded to a noticeable difference in the distance flown. Graphite pencils flew a mean distance of 33.06 inches while mechanical pencils flew a mean distance of 26.27 inches, about 13% less than graphite pencils. A t-test confirmed that the difference was statistically significant and confirmed my hypothesis. Additionally, both data distributions were mostly normal, except that graphite pencils were a little more bimodal and spread out.

A possible explanation for the difference in the mean distance flown is that graphite pencils are more uniform and their weight is more evenly distributed while mechanical pencils have a slightly heavier end on the eraser side. The large standard deviation and possible bimodality can be explained by variability in the experimental setup. Small differences in the speed and position of the roller used to launch the pencils can cause large differences in distance flown. While conducting the experiment, I noticed that on trials where the roller moved faster and hit the pencil closer to the end, the pencil generally traveled farther and vice versa. This phenomenon is part of sampling variability and cannot be prevented in practice, but caused the data to vary widely and even become slightly bimodal.

In future experiments, however, the roller can be arranged such that it is dropped and not rolled onto the pencil, which will minimize the variability. Also, a better measurement system should be used, as in this experiment, positioning the tape measure directly under the table was tricky. In addition, future experiments will need to be conducted to find the optimal pencil length, the amount of pencil over the edge, as well as whether using the eraser or non-eraser end of the pencil results in more distance flown.